Midterm solutions

① Fermi energy

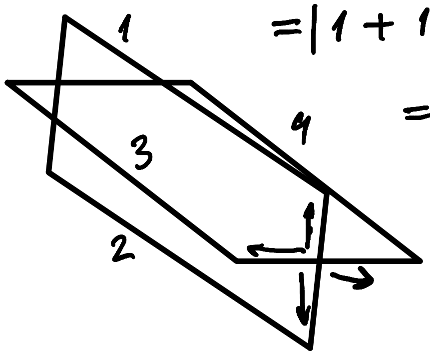
$$E_F = \frac{(3\pi^2 n^2)^{\frac{1}{3}} \hbar^2}{2m} = \frac{(3\pi^2)^{\frac{1}{3}} \hbar^2}{2 d^2 m} = 2.45 \text{ eV}$$

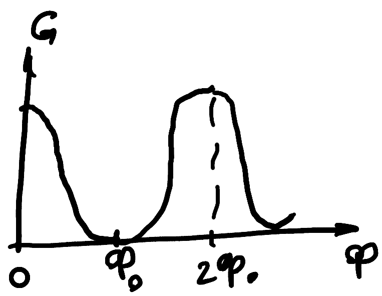
③

$$T = |e^{i\varphi_1} + e^{i\varphi_2} + e^{i\varphi_3} + e^{i\varphi_4}|^2$$

$$= |1 + 1 + e^{i\pi\frac{\varphi}{\varphi_0}} + e^{-i\pi\frac{\varphi}{\varphi_0}}|^2 =$$

$$= |2 + 2\cos(\pi\tfrac{\varphi}{\varphi_0})|^2$$





$$\varphi_0 = \frac{2\pi \hbar c}{e}$$

$(\varphi = BS)$

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④

$$\vec{E} = \vec{E}(t) = \int \frac{d\omega}{2\pi} \vec{E}(\omega)\, e^{-i\omega t}$$

$$\vec{j}(t) = \int \frac{d\omega}{2\pi} e^{-i\omega t} \vec{j}(\omega) = \int \frac{d\omega}{2\pi} \sigma(\omega) \vec{E}(\omega)\, e^{-i\omega t}$$

- allows one to find

when $\vec{E}(t) = \vec{\mathcal{E}}\,\delta(t)$, then $\vec{E}(\omega) = \vec{\mathcal{E}}$

$$\int \frac{d\omega}{2\pi} \frac{\sigma_0}{1 - i\omega\tau} \vec{\mathcal{E}}\, e^{-i\omega t} = \vec{\mathcal{E}}\, \sigma_0 \tau^{-1} e^{-\frac{t}{\tau}} \cdot \theta(t)$$

$$\boxed{\vec{j}(t) = \vec{\mathcal{E}}\, \sigma_0 \tau^{-1} e^{-\frac{t}{\tau}}\, \theta(t)}$$

Another method

Kinetic equation for $\vec{E}=0$

$$\frac{\partial f_p}{\partial t} = -\frac{f_p - f_p^0}{\tau}$$

(There are no gradient terms because $\vec{E}=0$ and everything is uniform)

$$f_p = f_p^0 + (f_p - f_p^0)\Big|_{t=0} \cdot e^{-\frac{t}{\tau}}$$

All modes relax with the same rate τ!

→ The whole current relaxes at that rate!

$$\vec{j}(t) = \vec{j}(0)\, e^{-\frac{t}{\tau}}$$

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(5) WL comes from the interference of paths on scales $L < L_\varphi$

In 2D $\quad \sigma = \frac{e^2}{4\pi\hbar} k_F l - \frac{e^2}{\pi^2\hbar} \ln\frac{L_\varphi}{l}$

Because L_φ has a power-law temperature dependence, it gives the temperature dependence of conductivity

$$\sigma \propto \text{const} - \ln T$$

The conductivity will be temperature independent when the magnetic length is shorter than L_φ

... the field is found from

is shorter than

Therefore, the field is found from the condition

$$B\, L_\varphi^2 \sim \Phi_0$$

$$B\, D\, \tau_\varphi \sim \Phi_0 \quad \rightarrow B \sim \frac{\Phi_0}{D\,\tau_\varphi} = \frac{\Phi_0 T^\alpha}{D a}$$